

## AIRY FUNCTIONS

## Table 10.11

| $x$  | $Ai(x)$     | $Ai'(x)$     | $Bi(x)$     | $Bi'(x)$    | $x$  | $Ai(x)$   | $Ai'(x)$  | $Bi(x)$   | $Bi'(x)$  |
|------|-------------|--------------|-------------|-------------|------|---|---|---|---|
| 0.00 | 0.35502 805 | -0.25881 940 | 0.61492 663 | 0.44828 836 | 0.50 | 0.23169 361   | -0.22491 053  | 0.85427 704   | 0.54457 256   |
| 0.01 | 0.35243 992 | -0.25880 174 | 0.61940 962 | 0.44831 926 | 0.51 | 0.22945 031   | -0.22374 617  | 0.85974 431   | 0.54890 049   |
| 0.02 | 0.34985 214 | -0.25874 909 | 0.62389 322 | 0.44841 254 | 0.52 | 0.22721 872   | -0.22257 027  | 0.86525 543   | 0.55334 239   |
| 0.03 | 0.34726 505 | -0.25866 197 | 0.62837 808 | 0.44856 911 | 0.53 | 0.22499 894   | -0.22138 322  | 0.87081 154   | 0.55789 959   |
| 0.04 | 0.34467 901 | -0.25854 090 | 0.63286 482 | 0.44878 987 | 0.54 | 0.22279 109   | -0.22018 541  | 0.87641 381   | 0.56257 345   |
| 0.05 | 0.34209 435 | -0.25838 640 | 0.63735 409 | 0.44907 570 | 0.55 | 0.22059 527   | -0.21897 720  | 0.88206 341   | 0.56736 532   |
| 0.06 | 0.33951 139 | -0.25819 898 | 0.64184 655 | 0.44942 752 | 0.56 | 0.21841 158   | -0.21775 898  | 0.88776 152   | 0.57227 662   |
| 0.07 | 0.33693 047 | -0.25797 916 | 0.64634 286 | 0.44984 622 | 0.57 | 0.21624 012   | -0.21653 112  | 0.89350 934   | 0.57730 873   |
| 0.08 | 0.33435 191 | -0.25772 745 | 0.65084 370 | 0.45033 270 | 0.58 | 0.21408 099   | -0.21529 397  | 0.89930 810   | 0.58246 311   |
| 0.09 | 0.33177 603 | -0.25744 437 | 0.65534 975 | 0.45088 787 | 0.59 | 0.21193 427   | -0.21404 790  | 0.90515 902   | 0.58774 120   |
| 0.10 | 0.32920 313 | -0.25713 042 | 0.65986 169 | 0.45151 263 | 0.60 | 0.20980 006   | -0.21279 326  | 0.91106 334   | 0.59314 448   |
| 0.11 | 0.32663 352 | -0.25678 613 | 0.66438 023 | 0.45220 789 | 0.61 | 0.20767 844   | -0.21153 041  | 0.91702 233   | 0.59867 447   |
| 0.12 | 0.32406 751 | -0.25644 200 | 0.66890 609 | 0.45297 457 | 0.62 | 0.20556 948   | -0.21025 970  | 0.92303 726   | 0.60433 267   |
| 0.13 | 0.32150 538 | -0.25600 854 | 0.67343 997 | 0.45381 357 | 0.63 | 0.20347 327   | -0.20898 146  | 0.92910 941   | 0.61012 064   |
| 0.14 | 0.31894 743 | -0.25557 625 | 0.67798 260 | 0.45472 582 | 0.64 | 0.20138 987   | -0.20769 605  | 0.93524 011   | 0.61603 997   |
| 0.15 | 0.31639 395 | -0.25511 565 | 0.68253 473 | 0.45571 223 | 0.65 | 0.19931 937   | -0.20640 378  | 0.94143 066   | 0.62209 226   |
| 0.16 | 0.31384 521 | -0.25462 724 | 0.68709 709 | 0.45677 373 | 0.66 | 0.19726 182   | -0.20510 500  | 0.94768 241   | 0.62827 912   |
| 0.17 | 0.31130 150 | -0.25411 151 | 0.69167 046 | 0.45791 125 | 0.67 | 0.19521 729   | -0.20380 004  | 0.95399 670   | 0.63460 222   |
| 0.18 | 0.30876 307 | -0.25356 898 | 0.69625 558 | 0.45912 572 | 0.68 | 0.19318 584   | -0.20248 920  | 0.96037 491   | 0.64106 324   |
| 0.19 | 0.30623 020 | -0.25300 013 | 0.70085 323 | 0.46041 808 | 0.69 | 0.19116 752   | -0.20117 281  | 0.96681 843   | 0.64766 389   |
| 0.20 | 0.30370 315 | -0.25240 547 | 0.70546 420 | 0.46178 928 | 0.70 | 0.18916 240   | -0.19985 119  | 0.97332 866   | 0.65440 592   |
| 0.21 | 0.30118 218 | -0.25178 548 | 0.71008 928 | 0.46324 026 | 0.71 | 0.18717 052   | -0.19852 464  | 0.97990 703   | 0.66129 109   |
| 0.22 | 0.29866 753 | -0.25114 067 | 0.71472 927 | 0.46477 197 | 0.72 | 0.18519 192   | -0.19719 347  | 0.98655 496   | 0.66832 121   |
| 0.23 | 0.29615 945 | -0.25047 151 | 0.71938 999 | 0.46638 539 | 0.73 | 0.18322 666   | -0.19585 798  | 0.99327 394   | 0.67549 810   |
| 0.24 | 0.29365 818 | -0.24977 850 | 0.72405 726 | 0.46808 147 | 0.74 | 0.18127 478   | -0.19451 846  | 1.00006 542   | 0.68282 363   |
| 0.25 | 0.29116 395 | -0.24906 211 | 0.72874 690 | 0.46986 119 | 0.75 | 0.17933 631   | -0.19317 521  | 1.00693 091   | 0.69029 970   |
| 0.26 | 0.28867 701 | -0.24832 284 | 0.73345 477 | 0.47172 554 | 0.76 | 0.17741 128   | -0.19182 851  | 1.01387 192   | 0.69792 824   |
| 0.27 | 0.28619 757 | -0.24756 115 | 0.73818 170 | 0.47367 549 | 0.77 | 0.17549 975   | -0.19047 865  | 1.02088 999   | 0.70571 121   |
| 0.28 | 0.28372 586 | -0.24677 753 | 0.74292 857 | 0.47571 205 | 0.78 | 0.17360 172   | -0.18912 591  | 1.02798 667   | 0.71365 062   |
| 0.29 | 0.28126 209 | -0.24597 244 | 0.74769 624 | 0.47783 623 | 0.79 | 0.17171 724   | -0.18777 055  | 1.03516 353   | 0.72174 849   |
| 0.30 | 0.27880 648 | -0.24514 636 | 0.75248 559 | 0.48004 903 | 0.80 | 0.16984 632   | -0.18641 286  | 1.04242 217   | 0.73000 690   |
| 0.31 | 0.27635 923 | -0.24439 976 | 0.75729 752 | 0.48235 148 | 0.81 | 0.16798 899   | -0.18505 310  | 1.04976 421   | 0.73842 795   |
| 0.32 | 0.27392 055 | -0.24363 309 | 0.76213 292 | 0.48474 462 | 0.82 | 0.16614 526   | -0.18369 153  | 1.05719 128   | 0.74701 380   |
| 0.33 | 0.27149 064 | -0.24285 682 | 0.76699 782 | 0.48722 948 | 0.83 | 0.16431 516   | -0.18232 840  | 1.06470 504   | 0.75576 663   |
| 0.34 | 0.26906 968 | -0.24164 140 | 0.77187 272 | 0.48980 713 | 0.84 | 0.16249 870   | -0.18096 398  | 1.07230 717   | 0.76468 865   |
| 0.35 | 0.26665 787 | -0.24071 730 | 0.77678 917 | 0.49247 861 | 0.85 | 0.16069 588   | -0.17959 851  | 1.07999 939   | 0.77378 215   |
| 0.36 | 0.26425 540 | -0.23977 495 | 0.78172 770 | 0.49524 501 | 0.86 | 0.15890 673   | -0.17823 223  | 1.08778 340   | 0.78304 942   |
| 0.37 | 0.26186 243 | -0.23881 481 | 0.78669 439 | 0.49810 741 | 0.87 | 0.15713 124   | -0.17686 539  | 1.09566 096   | 0.79249 282   |
| 0.38 | 0.25947 916 | -0.23783 731 | 0.79169 018 | 0.50106 692 | 0.88 | 0.15536 942   | -0.17549 823  | 1.10363 385   | 0.80211 473   |
| 0.39 | 0.25710 574 | -0.23684 291 | 0.79671 605 | 0.50412 463 | 0.89 | 0.15362 128   | -0.17413 097  | 1.11170 386   | 0.81191 759   |
| 0.40 | 0.25474 235 | -0.23583 203 | 0.80177 300 | 0.50728 168 | 0.90 | 0.15188 680   | -0.17276 384  | 1.11987 281   | 0.82190 389   |
| 0.41 | 0.25238 916 | -0.23480 512 | 0.80686 202 | 0.51053 920 | 0.91 | 0.15016 600   | -0.17139 708  | 1.12814 255   | 0.83207 615   |
| 0.42 | 0.25004 630 | -0.23376 259 | 0.81198 412 | 0.51389 833 | 0.92 | 0.14845 886   | -0.17003 090  | 1.13651 496   | 0.84243 695   |
| 0.43 | 0.24771 395 | -0.23270 487 | 0.81714 033 | 0.51736 025 | 0.93 | 0.14676 538   | -0.16866 551  | 1.14499 193   | 0.85298 891   |
| 0.44 | 0.24539 226 | -0.23163 239 | 0.82233 167 | 0.52092 614 | 0.94 | 0.14508 555   | -0.16730 113  | 1.15357 539   | 0.86373 470   |
| 0.45 | 0.24308 135 | -0.23054 556 | 0.82755 920 | 0.52459 717 | 0.95 | 0.14341 935   | -0.16593 797  | 1.16226 728   | 0.87467 704   |
| 0.46 | 0.24078 139 | -0.22944 479 | 0.83282 397 | 0.52837 457 | 0.96 | 0.14176 678   | -0.16457 623  | 1.17106 959   | 0.88581 871   |
| 0.47 | 0.23849 250 | -0.22833 050 | 0.83812 705 | 0.53225 956 | 0.97 | 0.14012 782   | -0.16321 611  | 1.17998 433   | 0.89716 253   |
| 0.48 | 0.23621 482 | -0.22720 310 | 0.84346 952 | 0.53625 338 | 0.98 | 0.13850 245   | -0.16185 781  | 1.18901 352   | 0.90871 137   |
| 0.49 | 0.23394 848 | -0.22606 297 | 0.84885 248 | 0.54035 729 | 0.99 | 0.13689 066   | -0.16050 153  | 1.19815 925   | 0.92046 818   |
| 0.50 | 0.23169 361 | -0.22491 053 | 0.85427 704 | 0.54457 256 | 1.00 | 0.13529 242   | -0.15914 744  | 1.20742 359   | 0.93243 593   |
|      |             |              |             |             |      | $\left[ \begin{smallmatrix} (-6)1 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)4 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)5 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)1 \\ 4 \end{smallmatrix} \right]$ |
|      |             |              |             |             |      | $\left[ \begin{smallmatrix} (-6)2 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)1 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)1 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)3 \\ 4 \end{smallmatrix} \right]$ |

## AIRY FUNCTIONS—AUXILIARY FUNCTIONS FOR LARGE POSITIVE ARGUMENTS

| $\xi^{-1}$ | $x$      | $f(-\xi)$ | $f(\xi)$ | $g(-\xi)$ | $g(\xi)$ | $\xi^{-1}$ | $x$      | $f(-\xi)$   | $f(\xi)$  | $g(-\xi)$   | $g(\xi)$  |
|------------|----------|-----------|----------|-----------|----------|------------|----------|---|---|---|---|
| 1.5        | 1.000000 | 0.527027  | 0.619912 | 0.619954  | 0.478728 | 0.50       | 2.080084 | 0.548230  | 0.593015  | 0.587245  | 0.526011  |
| 1.4        | 1.047069 | 0.528783  | 0.620335 | 0.617156  | 0.477925 | 0.45       | 2.231443 | 0.549584  | 0.589451  | 0.585235  | 0.530678  |
| 1.3        | 1.100099 | 0.530601  | 0.620327 | 0.614275  | 0.481658 | 0.40       | 2.413723 | 0.550980  | 0.585855  | 0.583174  | 0.535345  |
| 1.2        | 1.160397 | 0.532488  | 0.619799 | 0.611305  | 0.484018 | 0.35       | 2.638450 | 0.552421  | 0.582330  | 0.581056  | 0.539902  |
| 1.1        | 1.229700 | 0.534448  | 0.618649 | 0.608239  | 0.487107 | 0.30       | 2.924018 | 0.553912  | 0.578985  | 0.578878  | 0.544235  |
| 1.0        | 1.310371 | 0.536489  | 0.616764 | 0.605068  | 0.491037 | 0.25       | 3.301927 | 0.555456  | 0.575908  | 0.576635  | 0.548255  |
| 0.9        | 1.405721 | 0.538618  | 0.614022 | 0.601782  | 0.495921 | 0.20       | 3.831547 | 0.557058  | 0.573135  | 0.574320  | 0.551930  |
| 0.8        | 1.520550 | 0.540844  | 0.610309 | 0.598372  | 0.501859 | 0.15       | 4.641589 | 0.558724  | 0.570636  | 0.571927  | 0.555296  |
| 0.7        | 1.662119 | 0.543180  | 0.605543 | 0.594823  | 0.508909 | 0.10       | 6.082202 | 0.560462  | 0.568343  | 0.569448  | 0.558428  |
| 0.6        | 1.842016 | 0.545636  | 0.599723 | 0.591120  | 0.517032 | 0.05       | 9.654894 | 0.562280  | 0.566204  | 0.566873  | 0.561382  |
| 0.5        | 2.080084 | 0.548230  | 0.593015 | 0.587245  | 0.526011 | 0.00       | $\infty$ | 0.564190  | 0.564190  | 0.564190  | 0.564190  |
|            |          |           |          |           |          |            |          | $\left[ \begin{smallmatrix} (-5)1 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)4 \\ 6 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)1 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-5)4 \\ 6 \end{smallmatrix} \right]$ |

$$Ai(x) = \frac{1}{2} x^{-\frac{1}{2}} e^{-\xi} f(-\xi) \quad Bi(x) = x^{-\frac{1}{2}} e^{\xi} f(\xi) \quad Ai'(x) = -\frac{1}{2} x^{\frac{1}{2}} e^{-\xi} g(-\xi) \quad Bi'(x) = x^{\frac{1}{2}} e^{\xi} g(\xi) \quad \xi = \frac{2}{3} x^{\frac{3}{2}}$$

From J. C. P. Miller, The Airy integral, British Assoc. Adv. Sci. Mathematical Tables, Part-vol. B. Cambridge Univ. Press, Cambridge, England, 1946 (with permission).

Table 10.11

## AIRY FUNCTIONS

| $x$  | $Ai(-x)$  | $Ai'(-x)$   | $Bi(-x)$  | $Bi'(-x)$   | $x$  | $Ai(-x)$  | $Ai'(-x)$   | $Bi(-x)$  | $Bi'(-x)$   |
|------|---|---|---|---|------|---|---|---|---|
| 0.00 | 0.35502 805   | -0.25881 940  | 0.61492 663   | 0.44828 836   | 0.50 | 0.47572 809   | -0.20408 167  | 0.38035 266   | 0.50593 371   |
| 0.01 | 0.35761 619   | -0.25880 157  | 0.61044 364   | 0.44831 896   | 0.51 | 0.47775 692   | -0.20167 409  | 0.37528 379   | 0.50784 166   |
| 0.02 | 0.36020 397   | -0.25874 771  | 0.60596 005   | 0.44841 015   | 0.52 | 0.47976 138   | -0.19920 846  | 0.37019 579   | 0.50976 123   |
| 0.03 | 0.36279 102   | -0.25865 731  | 0.60147 524   | 0.44856 104   | 0.53 | 0.48174 089   | -0.19668 449  | 0.36508 853   | 0.51169 132   |
| 0.04 | 0.36537 699   | -0.25852 986  | 0.59698 863   | 0.44877 074   | 0.54 | 0.48369 487   | -0.19410 192  | 0.35996 193   | 0.51363 080   |
| 0.05 | 0.36796 149   | -0.25836 484  | 0.59249 963   | 0.44903 833   | 0.55 | 0.48562 274   | -0.19146 050  | 0.35481 589   | 0.51557 853   |
| 0.06 | 0.37054 416   | -0.25816 173  | 0.58800 767   | 0.44936 293   | 0.56 | 0.48752 389   | -0.18875 999  | 0.34965 033   | 0.51753 339   |
| 0.07 | 0.37312 460   | -0.25792 001  | 0.58351 218   | 0.44974 364   | 0.57 | 0.48939 774   | -0.18600 016  | 0.34446 520   | 0.51949 424   |
| 0.08 | 0.37570 243   | -0.25763 918  | 0.57901 261   | 0.45017 955   | 0.58 | 0.49124 369   | -0.18318 078  | 0.33926 043   | 0.52145 991   |
| 0.09 | 0.37827 725   | -0.25731 872  | 0.57450 841   | 0.45066 976   | 0.59 | 0.49306 115   | -0.18030 166  | 0.33403 599   | 0.52342 927   |
| 0.10 | 0.38084 867   | -0.25695 811  | 0.56999 904   | 0.45121 336   | 0.60 | 0.49484 953   | -0.17736 260  | 0.32879 184   | 0.52540 115   |
| 0.11 | 0.38341 628   | -0.25655 685  | 0.56548 397   | 0.45180 945   | 0.61 | 0.49660 821   | -0.17436 341  | 0.32352 796   | 0.52737 438   |
| 0.12 | 0.38597 967   | -0.25611 443  | 0.56096 268   | 0.45245 712   | 0.62 | 0.49833 659   | -0.17130 392  | 0.31824 435   | 0.52934 780   |
| 0.13 | 0.38853 843   | -0.25563 033  | 0.55643 466   | 0.45315 546   | 0.63 | 0.50003 408   | -0.16818 399  | 0.31294 101   | 0.53132 022   |
| 0.14 | 0.39109 213   | -0.25510 406  | 0.55189 940   | 0.45390 355   | 0.64 | 0.50170 007   | -0.16500 345  | 0.30761 795   | 0.53329 046   |
| 0.15 | 0.39364 037   | -0.25453 511  | 0.54735 642   | 0.45470 047   | 0.65 | 0.50333 395   | -0.16176 218  | 0.30227 521   | 0.53525 733   |
| 0.16 | 0.39618 269   | -0.25392 297  | 0.54280 523   | 0.45554 530   | 0.66 | 0.50493 511   | -0.15846 007  | 0.29691 282   | 0.53721 964   |
| 0.17 | 0.39871 868   | -0.25326 716  | 0.53824 536   | 0.45643 713   | 0.67 | 0.50650 295   | -0.15509 701  | 0.29153 084   | 0.53917 618   |
| 0.18 | 0.40124 789   | -0.25256 716  | 0.53367 634   | 0.45737 503   | 0.68 | 0.50803 685   | -0.15167 290  | 0.28612 932   | 0.54112 575   |
| 0.19 | 0.40376 987   | -0.25182 250  | 0.52909 771   | 0.45835 806   | 0.69 | 0.50953 620   | -0.14818 768  | 0.28070 835   | 0.54306 714   |
| 0.20 | 0.40628 419   | -0.25103 267  | 0.52450 903   | 0.45938 529   | 0.70 | 0.51100 040   | -0.14464 129  | 0.27526 801   | 0.54499 912   |
| 0.21 | 0.40879 038   | -0.25019 720  | 0.51990 986   | 0.46045 578   | 0.71 | 0.51242 882   | -0.14103 366  | 0.26980 840   | 0.54692 048   |
| 0.22 | 0.41128 798   | -0.24931 559  | 0.51529 977   | 0.46156 860   | 0.72 | 0.51382 087   | -0.13736 479  | 0.26432 964   | 0.54883 000   |
| 0.23 | 0.41377 653   | -0.24838 737  | 0.51067 835   | 0.46272 279   | 0.73 | 0.51517 591   | -0.13363 464  | 0.25883 185   | 0.55072 642   |
| 0.24 | 0.41625 557   | -0.24741 206  | 0.50604 518   | 0.46391 740   | 0.74 | 0.51649 336   | -0.12984 322  | 0.25331 516   | 0.55260 852   |
| 0.25 | 0.41872 461   | -0.24638 919  | 0.50139 987   | 0.46515 148   | 0.75 | 0.51777 258   | -0.12599 055  | 0.24777 973   | 0.55447 506   |
| 0.26 | 0.42118 319   | -0.24531 828  | 0.49674 203   | 0.46642 408   | 0.76 | 0.51901 296   | -0.12207 665  | 0.24222 571   | 0.55632 480   |
| 0.27 | 0.42363 082   | -0.24419 888  | 0.49207 127   | 0.46773 423   | 0.77 | 0.52021 390   | -0.11810 157  | 0.23665 329   | 0.55815 647   |
| 0.28 | 0.42606 701   | -0.24303 053  | 0.48738 722   | 0.46908 095   | 0.78 | 0.52137 479   | -0.11406 538  | 0.23106 265   | 0.55996 884   |
| 0.29 | 0.42849 126   | -0.24181 276  | 0.48268 953   | 0.47046 327   | 0.79 | 0.52249 501   | -0.10996 815  | 0.22545 398   | 0.56176 063   |
| 0.30 | 0.43090 310   | -0.24054 513  | 0.47797 784   | 0.47188 022   | 0.80 | 0.52357 395   | -0.10580 999  | 0.21982 751   | 0.56353 059   |
| 0.31 | 0.43330 200   | -0.23922 719  | 0.47325 181   | 0.47333 081   | 0.81 | 0.52461 101   | -0.10159 101  | 0.21418 345   | 0.56527 745   |
| 0.32 | 0.43568 747   | -0.23785 851  | 0.46851 112   | 0.47481 405   | 0.82 | 0.52560 557   | -0.09731 134  | 0.20852 204   | 0.56699 994   |
| 0.33 | 0.43805 900   | -0.23643 865  | 0.46375 543   | 0.47632 895   | 0.83 | 0.52655 703   | -0.09297 113  | 0.20284 354   | 0.56869 679   |
| 0.34 | 0.44041 607   | -0.23496 718  | 0.45898 443   | 0.47787 450   | 0.84 | 0.52746 479   | -0.08857 055  | 0.19714 820   | 0.57036 671   |
| 0.35 | 0.44275 817   | -0.23344 368  | 0.45419 784   | 0.47944 970   | 0.85 | 0.52832 824   | -0.08410 979  | 0.19143 630   | 0.57200 845   |
| 0.36 | 0.44508 477   | -0.23186 773  | 0.44939 534   | 0.48105 354   | 0.86 | 0.52914 678   | -0.07958 904  | 0.18570 813   | 0.57362 071   |
| 0.37 | 0.44739 535   | -0.23023 893  | 0.44457 667   | 0.48268 500   | 0.87 | 0.52991 982   | -0.07500 854  | 0.17996 399   | 0.57520 220   |
| 0.38 | 0.44968 937   | -0.22855 687  | 0.43974 156   | 0.48434 307   | 0.88 | 0.53064 676   | -0.07036 852  | 0.17420 419   | 0.57675 165   |
| 0.39 | 0.45196 631   | -0.22682 116  | 0.43488 973   | 0.48602 670   | 0.89 | 0.53132 700   | -0.06566 925  | 0.16842 906   | 0.57826 777   |
| 0.40 | 0.45422 561   | -0.22503 141  | 0.43002 094   | 0.48773 486   | 0.90 | 0.53195 995   | -0.06091 100  | 0.16263 895   | 0.57974 926   |
| 0.41 | 0.45646 675   | -0.22318 723  | 0.42513 495   | 0.48946 652   | 0.91 | 0.53254 502   | -0.05609 407  | 0.15683 420   | 0.58119 484   |
| 0.42 | 0.45868 678   | -0.22128 826  | 0.42023 153   | 0.49122 062   | 0.92 | 0.53308 163   | -0.05121 879  | 0.15101 518   | 0.58260 321   |
| 0.43 | 0.46089 233   | -0.21933 412  | 0.41531 047   | 0.49299 611   | 0.93 | 0.53356 920   | -0.04628 549  | 0.14518 226   | 0.58397 309   |
| 0.44 | 0.46307 567   | -0.21732 447  | 0.41037 154   | 0.49479 193   | 0.94 | 0.53400 715   | -0.04129 452  | 0.13933 585   | 0.58530 317   |
| 0.45 | 0.46523 864   | -0.21525 894  | 0.40541 457   | 0.49660 702   | 0.95 | 0.53439 490   | -0.03624 628  | 0.13347 634   | 0.58659 217   |
| 0.46 | 0.46738 066   | -0.21313 721  | 0.40043 934   | 0.49844 031   | 0.96 | 0.53473 189   | -0.03114 116  | 0.12760 415   | 0.58783 879   |
| 0.47 | 0.46950 119   | -0.21095 893  | 0.39544 570   | 0.50029 070   | 0.97 | 0.53501 754   | -0.02597 957  | 0.12171 971   | 0.58904 173   |
| 0.48 | 0.47159 965   | -0.20872 379  | 0.39043 348   | 0.50215 713   | 0.98 | 0.53525 129   | -0.02076 197  | 0.11582 346   | 0.59019 974   |
| 0.49 | 0.47367 548   | -0.20643 147  | 0.38540 251   | 0.50403 850   | 0.99 | 0.53543 259   | -0.01548 880  | 0.10991 587   | 0.59131 145   |
| 0.50 | 0.47572 809   | -0.20408 167  | 0.38035 266   | 0.50593 371   | 1.00 | 0.53556 088   | -0.01016 057  | 0.10399 739   | 0.59237 563   |
|      | $\left[ \begin{smallmatrix} (-6)3 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)7 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)2 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)8 \\ 4 \end{smallmatrix} \right]$ |      | $\left[ \begin{smallmatrix} (-6)7 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)8 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)2 \\ 4 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-6)6 \\ 4 \end{smallmatrix} \right]$ |

## AIRY FUNCTIONS

Table 10.11

| $x$ | $\text{Ai}(-x)$   | $\text{Ai}'(-x)$  | $\text{Bi}(-x)$   | $\text{Bi}'(-x)$  | $x$  | $\text{Ai}(-x)$   | $\text{Ai}'(-x)$   | $\text{Bi}(-x)$   | $\text{Bi}'(-x)$   |
|-----|---|---|---|---|------|---|--|---|--|
| 1.0 | 0.53556 088   | -0.01016 057  | +0.10399 739  | 0.59237 563   | 5.5  | +0.01778 154  | 0.86419 722  | -0.36781 345  | +0.02511 158   |
| 1.1 | 0.53381 051   | +0.04602 915  | +0.04432 659  | 0.60011 970   | 5.6  | -0.06833 070  | 0.85003 256  | -0.36017 223  | -0.17783 760   |
| 1.2 | 0.52619 437   | 0.10703 157   | -0.01582 137  | 0.60171 016   | 5.7  | -0.15062 016  | 0.78781 722  | -0.33245 825  | -0.37440 903   |
| 1.3 | 0.51227 201   | 0.17199 181   | -0.07576 964  | 0.59592 975   | 5.8  | -0.22435 192  | 0.67943 152  | -0.28589 021  | -0.55300 203   |
| 1.4 | 0.49170 018   | 0.23981 912   | -0.13472 406  | 0.58165 624   | 5.9  | -0.28512 278  | 0.52962 857  | -0.22282 969  | -0.70247 952   |
| 1.5 | 0.46425 658   | 0.30918 697   | -0.19178 486  | 0.55790 810   | 6.0  | -0.32914 517  | 0.34593 549  | -0.14669 838  | -0.81289 879   |
| 1.6 | 0.42986 298   | 0.37854 219   | -0.24596 320  | 0.52389 354   | 6.1  | -0.35351 168  | +0.13836 394   | -0.06182 255  | -0.87622 530   |
| 1.7 | 0.38860 704   | 0.44612 455   | -0.29620 266  | 0.47906 134   | 6.2  | -0.35642 107  | -0.08106 856   | +0.02679 081  | -0.88697 896   |
| 1.8 | 0.34076 156   | 0.50999 763   | -0.34140 583  | 0.42315 137   | 6.3  | -0.33734 765  | -0.29899 161   | 0.11373 701   | -0.84276 110   |
| 1.9 | 0.28680 006   | 0.56809 172   | -0.38046 588  | 0.35624 251   | 6.4  | -0.29713 762  | -0.50147 985   | 0.19354 136   | -0.74461 387   |
| 2.0 | 0.22740 743   | 0.61825 902   | -0.41230 259  | 0.27879 517   | 6.5  | -0.23802 030  | -0.67495 249   | 0.26101 266   | -0.59717 067   |
| 2.1 | 0.16348 451   | 0.65834 069   | -0.43590 235  | 0.19168 563   | 6.6  | -0.16352 646  | -0.80711 925   | 0.31159 995   | -0.40856 734   |
| 2.2 | 0.09614 538   | 0.68624 482   | -0.45036 098  | +0.09622 919  | 6.7  | -0.07831 247  | -0.88790 797   | 0.34172 774   | -0.19009 878   |
| 2.3 | +0.02670 633  | 0.70003 366   | -0.45492 823  | -0.00581 106  | 6.8  | +0.01210 452  | -0.91030 401   | 0.34908 418   | +0.04437 678   |
| 2.4 | -0.04333 414  | 0.69801 760   | -0.44905 228  | -0.11223 237  | 6.9  | 0.10168 800   | -0.87103 106   | 0.33283 784   | 0.27926 391  |
| 2.5 | -0.11232 507  | 0.67885 273   | -0.43242 247  | -0.22042 015  | 7.0  | 0.18428 084   | -0.77100 817   | 0.29376 207   | 0.49824 459  |
| 2.6 | -0.17850 243  | 0.64163 799   | -0.40500 828  | -0.32739 717  | 7.1  | 0.25403 633   | -0.61552 879   | 0.23425 088   | 0.68542 058  |
| 2.7 | -0.24003 811  | 0.58600 720   | -0.36709 211  | -0.42989 534  | 7.2  | 0.30585 152   | -0.41412 428   | 0.15821 739   | 0.82650 634  |
| 2.8 | -0.29509 759  | 0.51221 098   | -0.31929 389  | -0.52445 040  | 7.3  | 0.33577 037   | -0.18009 580   | +0.07087 411  | 0.90998 427  |
| 2.9 | -0.34190 510  | 0.42118 281   | -0.26258 500  | -0.60751 829  | 7.4  | 0.34132 375   | +0.07027 632   | -0.02159 652  | 0.92812 809  |
| 3.0 | -0.37881 429  | 0.31458 377   | -0.19828 963  | -0.67561 122  | 7.5  | 0.32177 572   | 0.31880 951  | -0.11246 349  | 0.87780 228  |
| 3.1 | -0.40438 222  | 0.19482 045   | -0.12807 165  | -0.72544 957  | 7.6  | 0.27825 023   | 0.54671 882  | -0.19493 376  | 0.76095 509  |
| 3.2 | -0.41744 342  | +0.06503 115  | -0.05390 576  | -0.75412 455  | 7.7  | 0.21372 037   | 0.73605 242  | -0.26267 007  | 0.58474 045  |
| 3.3 | -0.41718 094  | -0.07096 362  | +0.02196 800  | -0.75926 518  | 7.8  | 0.13285 154   | 0.87115 540  | -0.31030 057  | 0.36122 930  |
| 3.4 | -0.40319 048  | -0.20874 905  | 0.09710 619   | -0.73920 163  | 7.9  | +0.04170 188  | 0.94004 300  | -0.33387 856  | +0.10670 215   |
| 3.5 | -0.37553 382  | -0.34344 343  | 0.16893 984   | -0.69311 628  | 8.0  | -0.05270 505  | 0.93556 094  | -0.33125 158  | -0.15945 050   |
| 3.6 | -0.33477 748  | -0.46986 397  | 0.23486 631   | -0.62117 283  | 8.1  | -0.14290 815  | 0.85621 859  | -0.30230 331  | -0.41615 664   |
| 3.7 | -0.28201 306  | -0.58272 780  | 0.29235 261   | -0.52461 361  | 8.2  | -0.22159 945  | 0.70659 870  | -0.24904 019  | -0.64232 293   |
| 3.8 | -0.21885 598  | -0.67688 257  | 0.33904 647   | -0.40581 592  | 8.3  | -0.28223 176  | 0.49727 679  | -0.17550 556  | -0.81860 044   |
| 3.9 | -0.14741 991  | -0.74755 809  | 0.37289 058   | -0.26829 836  | 8.4  | -0.31959 219  | +0.24422 089   | -0.08751 798  | -0.92910 958   |
| 4.0 | -0.07026 553  | -0.79062 858  | 0.39223 471   | -0.11667 057  | 8.5  | -0.33029 024  | -0.03231 335   | +0.00775 444  | -0.96296 917   |
| 4.1 | +0.00967 698  | -0.80287 254  | 0.39593 974   | +0.04347 872  | 8.6  | -0.31311 245  | -0.30933 027   | 0.10235 647   | -0.91547 918   |
| 4.2 | 0.08921 076   | -0.78221 561  | 0.38346 736   | 0.20575 691   | 8.7  | -0.26920 454  | -0.56297 685   | 0.18820 363   | -0.78882 623   |
| 4.3 | 0.16499 781   | -0.72794 081  | 0.35494 906   | 0.36320 468   | 8.8  | -0.20205 445  | -0.77061 301   | 0.25778 240   | -0.59221 371   |
| 4.4 | 0.23370 326   | -0.64085 018  | 0.31122 860   | 0.50858 932   | 8.9  | -0.11726 631  | -0.91289 276   | 0.30483 241   | -0.34136 475   |
| 4.5 | 0.29215 278   | -0.52336 253  | 0.25387 266   | 0.63474 477   | 9.0  | -0.02213 372  | -0.97566 398   | 0.32494 732   | -0.05740 051   |
| 4.6 | 0.33749 598   | -0.37953 391  | 0.18514 576   | 0.73494 444   | 9.1  | +0.07495 989  | -0.95149 682   | 0.31603 471   | +0.23484 379   |
| 4.7 | 0.36736 748   | -0.21499 018  | 0.10794 695   | 0.80328 926   | 9.2  | 0.16526 800   | -0.84067 107   | 0.27858 425   | 0.50894 402  |
| 4.8 | 0.38003 668   | -0.03676 510  | +0.02570 779  | 0.83508 976   | 9.3  | 0.24047 380   | -0.65149 241   | 0.21570 835   | 0.73928 028  |
| 4.9 | 0.37453 635   | +0.14695 743  | -0.05774 655  | 0.82721 903   | 9.4  | 0.29347 756   | -0.39986 237   | 0.13293 876   | 0.90348 537  |
| 5.0 | 0.35076 101   | 0.32719 282   | -0.13836 913  | 0.77841 177   | 9.5  | 0.31910 325   | -0.10809 532   | +0.03778 543  | 0.98471 407  |
| 5.1 | 0.30952 600   | 0.49458 600   | -0.21208 913  | 0.68948 513   | 9.6  | 0.31465 158   | +0.19695 044   | -0.06091 293  | 0.97349 918  |
| 5.2 | 0.25258 034   | 0.63990 517   | -0.27502 704  | 0.56345 898   | 9.7  | 0.28023 750   | 0.48628 629  | -0.15379 421  | 0.86898 388  |
| 5.3 | 0.18256 793   | 0.75457 542   | -0.32371 608  | 0.40555 694   | 9.8  | 0.21886 743   | 0.73154 486  | -0.23186 331  | 0.67936 774  |
| 5.4 | 0.10293 460   | 0.83122 307   | -0.35531 708  | 0.22307 496   | 9.9  | 0.13623 503   | 0.90781 333  | -0.28738 356  | 0.42147 209  |
| 5.5 | 0.01778 154   | 0.86419 722   | -0.36781 345  | 0.02511 158   | 10.0 | 0.04024 124   | 0.99626 504  | -0.31467 983  | 0.11941 411  |
|     | $\left[ \begin{smallmatrix} (-3)2 \\ 8 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-3)5 \\ 8 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-3)2 \\ 8 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-3)5 \\ 9 \end{smallmatrix} \right]$ |      | $\left[ \begin{smallmatrix} (-3)4 \\ 9 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-2)1 \\ 10 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-3)4 \\ 9 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-2)1 \\ 10 \end{smallmatrix} \right]$ |

## AIRY FUNCTIONS—AUXILIARY FUNCTIONS FOR LARGE NEGATIVE ARGUMENTS

| $\tau^{-1}$ | $x$       | $f_1(\tau)$ | $f_2(\tau)$ | $g_1(\tau)$ | $g_2(\tau)$ | $\langle \tau \rangle$ |
|-------------|-----------|-------------|-------------|-------------|-------------|------------------------|
| 0.05        | 9.654894  | 0.39752 21  | 0.40028 87  | 0.40092 31  | 0.39704 87  | 20                     |
| 0.04        | 11.203512 | 0.39781 14  | 0.40002 58  | 0.40052 06  | 0.39741 99  | 25                     |
| 0.03        | 13.572088 | 0.39809 83  | 0.39975 97  | 0.40012 11  | 0.39779 49  | 33                     |
| 0.02        | 17.784467 | 0.39838 24  | 0.39949 03  | 0.39972 48  | 0.39817 37  | 50                     |
| 0.01        | 28.231081 | 0.39866 38  | 0.39921 79  | 0.39933 19  | 0.39855 62  | 100                    |

|      |          |   |   |   |   |          |
|------|----------|---|---|---|---|----------|
| 0.00 | $\infty$ | 0.39894 23  | 0.39894 23  | 0.39894 23  | 0.39894 23  | $\infty$ |
|      |          | $\left[ \begin{smallmatrix} (-7)4 \\ 3 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-7)4 \\ 3 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-7)4 \\ 3 \end{smallmatrix} \right]$ | $\left[ \begin{smallmatrix} (-7)5 \\ 3 \end{smallmatrix} \right]$ |          |

$$\text{Ai}(-x) = x^{-\frac{1}{4}} [f_1(\tau) \cos \tau + f_2(\tau) \sin \tau] \quad \text{Bi}(-x) = x^{-\frac{1}{4}} [f_2(\tau) \cos \tau - f_1(\tau) \sin \tau]$$

$$\text{Ai}'(-x) = x^{\frac{1}{4}} [g_1(\tau) \sin \tau - g_2(\tau) \cos \tau] \quad \text{Bi}'(-x) = x^{\frac{1}{4}} [g_1(\tau) \cos \tau + g_2(\tau) \sin \tau]$$

$$\tau = \frac{2}{3} x^{\frac{3}{2}}$$

$\langle \tau \rangle$  = nearest integer to  $\tau$ .